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Estimation Of Exponential Mean Life In Complete And Failure Censored Samples With Prior Information

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ABSTRACT

A family of estimators for exponential mean life is proposed when a guessed value and a guessed interval of mean life are available in addition to sample information. Shrinkage framework has been used to improve the uniformly minimum variance unbiased estimator (UMVUE). Conjugate gamma prior is considered to derive a Bayes estimator and this estimate has been employed in shrinkage estimation to resolve the choice of the natural origin towards which the UMVUE to be shrunken. The suggested family of estimators has been compared with the classical estimators in terms of percent relative efficiency. Subsequently, applicability of the proposed family of estimators has been tested by a life-testing example.

Key Words & Phrases: Exponential distribution, Prior information, Shrinkage estimation, Bayesian Estimation, Bias, Mean squared error.

1. INTRODUCTION

The exponential distribution is a very commonly used distribution in reliability engineering. Due to its simplicity, it has been widely employed even in cases to which it does not apply. Exponential distribution plays an important part in life testing problems too. For a situation where the failure rate appears to be more or less constant, the exponential distribution would be an adequate choice. Davis (1952) examined different types of data and the exponential distribution appears to fit most of the situations quite well. The probability density function of one-parameter exponential distribution is given by:

$$f(x;\theta) = \frac{1}{\theta} \exp\left(\frac{-x}{\theta}\right) \;;\; x \geq 0,\; \theta > 0, \tag{1.1}$$

where θ is the average life of the item and it acts as a scale parameter.

Suppose n items are subjected to test and the test is terminated after all the items have failed. The samples thus obtained are called 'failure-complete' samples. However, there are several situations where this is neither possible nor desirable. Note that life-testing experiments are usually destructive in nature that the items are destroyed at the end of the experiment and cannot be used again. This limits the number of items we can test. We may put n items on test and terminate the experiment when a pre-assigned number of items, say r ($< n$) have failed. The samples obtained from such an experiment are called 'failure-censored' samples. Failure-censored sampling is almost mandatory in dealing with high cost sophisticated items such as colour television tubes. In this case the data consist of the life times of r items that failed (say $x_{(1)} < x_{(2)} < \ldots < x_{(r)}$) and the fact that $(n - r)$ items have survived beyond $x_{(r)}$. Here, the number of items that failed, r, is fixed while $x_{(r)}$, the time at

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which the experiment is terminated, is a random variable. The classical estimators of θ in complete and censored sampling are summarized in Table 1.1 with their characteristics.

Table 1.1. Classical Estimators of Mean Life θ

Type of Sample	Type of Estimator	Estimator	Absolute Relative Bias	Mean Squared Error
Complete	Unbiased	$\tilde{\theta}_U = \bar{x}$	---	$\text{Var}(\tilde{\theta}_U) = (\theta^2/n)$
	MMSE	$\tilde{\theta}_M = n\bar{x}/(n+1)$	$\text{ARB}(\tilde{\theta}_M) = 1/(n+1)$	$\text{MSE}(\tilde{\theta}_M) = \theta^2/(n+1)$
Censored	Unbiased	$\tilde{\tilde{\theta}}_U = (S_r/r)$	---	$\text{Var}(\tilde{\tilde{\theta}}_U) = (\theta^2/r)$
	MMSE	$\tilde{\tilde{\theta}}_M = [S_r/(r+1)]$	$\text{ARB}(\tilde{\tilde{\theta}}_M) = 1/(r+1)$	$\text{MSE}(\tilde{\tilde{\theta}}_M) = \theta^2/(r+1)$

where, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $S_r = \left[\sum_{i=1}^{r} x_i + (n-r)x_r\right]$

It is apparent that the above estimators take into account the sample information alone. However, Davis and Arnold (1970) have shown that, in terms of squared error risk, the usual unbiased estimator should not necessarily be considered. They have exhibited that one can improve upon the unique best mean squared error estimator. It is normally believed that due to considerable handling of life-times of an item in the past, one may have some prior information about the scale parameter θ in the form of either a point value, an interval or in the form of a prior distribution.

Bayesian and Shrinkage estimation techniques are the well-known estimation procedures that use prior information. Kotz et. al. (1988) defined shrinkage (shrunken) estimator as an estimator obtained through modification of the usual (maximum likelihood, minimum variance unbiased, least squares etc.) estimator in order to optimize some desirable criterion function like mean squared error (MSE), quadratic risk, bias etc. If θ_0 is an educated guess or an initial estimate of the value of the parameter θ then in such cases it may be reasonable to take the usual estimator for θ, say $\tilde{\theta}_U$, and move it closer to (or shrink it toward) this so-called natural origin θ_0 by multiplying the difference $(\tilde{\theta}_U - \theta_0)$ by a shrinking factor k and adding it to θ_0, i.e.,

$$\hat{\theta}_S = k(\tilde{\theta}_U - \theta_0) + \theta_0 = k\tilde{\theta}_U + (1-k)\theta_0, \quad 0 \le k \le 1 \qquad \text{(cf., Thompson (1968 a)}$$

The resulting estimator, though perhaps biased, has a smaller MSE than $\tilde{\theta}_U$ for θ in some interval around θ_0 (the so-called effective interval). Sometimes instead of guessing the value θ_0 of θ it may be more reasonable to shrink towards an interval, i.e., guess an interval (θ_1, θ_2) that we believe contains θ; cf. Lemmer (1981) and Thompson (1968 b). A lot many shrinkage estimators have been reported by various authors including Pandey and Singh (1983), Pandey and Srivastava (1985), Jani (1991), Singh et. al. (1993), Singh and Raghuvanshi (1996), Upadhyaya et. al. (1997), Singh et. al. (2001) and Singh and Saxena

(2001) for exponential mean life. In all the shrinkage estimators suggested so far, the usual estimator $\tilde{\theta}_U$ is shrunken towards a guessed value θ_0 or an interval (θ_1, θ_2).

This paper considers a situation where life tester possesses prior information in the form of a guessed value and a guessed interval both. The situation dealt with the case when life tester does not have much confidence to express his belief just by one value as a prior estimate of the parameter i.e., the point guessed value, however he is in a position to express his belief strongly in the form of a certain range i.e., the guessed interval. Apparently it is assumed that the point guessed value falls within the guessed interval, i.e., $\theta_1 < \theta_0 < \theta_2$. It is obvious to question that what will be the natural origin if guessed value and guessed interval both to be incorporated in the shrinkage estimation procedure? Can we really shrink simultaneously towards a point and an interval? An appropriate approach may be to shrink towards a point, say θ_B, in which the knowledge available in the form of θ_0, θ_1 and θ_2 must be integrated. This problem can be solved in Bayesian framework.

2. BAYESIAN APPROACH TO DETERMINE THE NATURAL ORIGIN

Consider the model (1.1) as

$$f(x \mid \eta) = \eta \exp(-\eta x) \quad ; x \geq 0, \ \eta > 0$$

where $\eta = (1 / \theta)$. The standard argument given by Raifa and Schlaifer (1961) leads to select a conjugate prior

$$g(\eta) = \frac{\alpha^{\beta} \eta^{\beta-1} \exp(-\alpha \eta)}{\Gamma(\beta)} \ ; \alpha > 0, \ \beta > 0, \ 0 < \eta < \infty$$

which is a Gamma (α, β) distribution with mean and variance given by $E(\eta) = (\beta / \alpha) = \mu$ (say) and $Var(\eta) = (\beta / \alpha^2) = (\mu / \alpha)$ respectively. Under the assumption of prior density $g(\eta)$, the posterior distribution of η can be obtained by using the Baye's theorem as

$$g^{*}(\eta \mid x) = \frac{1}{\Gamma(\beta + n)} \left(\alpha + \sum_{i=1}^{n} x_i \right)^{\beta+n} \eta^{\beta+n-1} \exp\left\{ -\eta \left(\alpha + \sum_{i=1}^{n} x_i \right) \right\} ; \alpha > 0, \ \beta > 0, \ 0 < \eta < \infty$$

which is a $\text{Gamma}(\alpha + \sum x_i, \ \beta + n)$ distribution involves two hyperparameters to be assessed namely, α and β. A very crude possible way of selecting these parameters involves: let, $\eta_j = (1/\theta_j)$; $j = 0, 1, 2$. The value of μ and α can be obtained by simultaneously solving the two equations:

$$\mu + \xi_1 \sqrt{\frac{\mu}{\alpha}} = \eta_1 \quad \text{and} \quad \mu - \xi_2 \sqrt{\frac{\mu}{\alpha}} = \eta_2 \quad ; \text{if} \quad \theta_0 < \theta_{mid}$$

or $\mu - \xi_1\sqrt{\frac{\mu}{\alpha}} = \eta_1$ and $\mu \div \xi_2\sqrt{\frac{\mu}{\alpha}} = \eta_2$; if $\theta_0 > \theta_{mid}$

where $\xi_1 = \frac{6(\eta_1 - \eta_0)}{\eta_1 - \eta_2}$ and $\xi_2 = \frac{6(\eta_0 - \eta_2)}{\eta_1 - \eta_2}$ are the weights based on the area property of the standard normal distribution, $\theta_{mid} = (\theta_1 + \theta_2)/2$. Once one gets the values of μ and α, it is trivial to calculate β. Thus, the numerical value of posterior mean of η is given by $\eta_B = \frac{\beta + n}{\alpha + \sum x_i}$ and finally, a Bayesian estimate of θ is given by $\theta_B = (1/\eta_B)$. In the next section, this Bayes estimate has been used as a natural origin towards which the UMVUE has been shrunken by optimizing the mean squared error.

3. SHRINKAGE ESTIMATION WITH COMPLETE SAMPLES

Jani (1991) suggested a class of shrinkage estimators when a prior point estimate θ_0 of θ is available viz.,

$$\tilde{\theta}_{(p)} = \theta_0\left[1 + W\left(\frac{\theta_0}{\bar{x}}\right)^p\right].$$

Motivated by Jani (1991) we evoked a family of estimators $\tilde{\theta}_{(p,q)}$ for θ defined in (3.1). The idea behind this type of estimator is that one's knowledge of θ can best be expressed by mean of a prior distribution of θ around the values θ_0, θ_1 and θ_2 rather than a one-point distribution in θ_0. Furthermore, an additional scalar q would give sufficient scope to control mean squared error. Thus we suggest a Bayesian-shrinkage family of estimators as

$$\tilde{\theta}_{(p,q)} = \theta_B\left[q + W\left(\frac{\theta_B}{\bar{x}}\right)^p\right], \tag{3.1}$$

where p and q are real numbers such that $p \neq 0$ and $0 < q < \infty$, W is a stochastic variable which may in particular be a scalar to be chosen such that the mean squared error (MSE) of $\tilde{\theta}_{(p,q)}$ is minimum and θ_B is a Bayes estimate to be determined as discussed in the previous section.

Assuming W as a scalar, the MSE of $\tilde{\theta}_{(p,q)}$ is obtained as

$$\mathrm{MSE}(\tilde{\theta}_{p,q}) = \theta^2\left[(q\lambda - 1)^2 + \frac{W^2 n^{2p}\lambda^{2(p+1)}\Gamma(n-2p)}{\Gamma(n)} + \frac{2(q\lambda - 1)W n^p \lambda^{(p+1)}\Gamma(n-p)}{\Gamma(n)}\right]$$

where $\lambda = \theta_B/\theta$. Minimizing with respect to W and replacing θ by $\tilde{\theta}_U$, gives

$$\hat{W} = \frac{-[q\,\theta_B - \bar{x}]\,\bar{x}^p}{\theta_B^{p+1}} W_{(n,p)}$$

where $W_{(n,p)} = \frac{\Gamma(n-p)}{n^p\,\Gamma(n-2p)}$ lies between 0 and 1, i.e., $0 < W_{(n,p)} \le 1$ provided gamma functions exist, i.e., $p < (n/2)$. Now, substituting $\hat{W}$ in (3.1) yields a class of shrinkage estimators for θ in a more feasible form as

$$\hat{\theta}_{(p,q)} = \bar{x}\;W_{(n,p)} + q\;\theta_B\left(1 - W_{(n,p)}\right) \tag{3.2}$$

Clearly, this is the convex combination of $\bar{x}$ and $q\,\theta_B$, hence $\hat{\theta}_{(p,q)}$ is always positive. If $W_{(n,p)} = 1$, the proposed class of shrinkage estimators in (3.2) turns into the uniformly minimum variance unbiased estimator (UMVUE) $\tilde{\theta}_U$, otherwise it is biased, the absolute relative bias of which is given by

$$\text{ARB}\left\{\hat{\theta}_{(p,q)}\right\} = \left|\{q\lambda - 1\}\left(1 - W_{(n,p)}\right)\right|.$$

The condition for unbiasedness of $\hat{\theta}_{(p,q)}$ that $W_{(n,p)} = 1$, holds *if and only if*, sample size n is indefinitely large, i.e., $n \to \infty$. Moreover, if the proposed class of estimators $\hat{\theta}_{(p,q)}$ turns into $\tilde{\theta}_U$ then this case does not deal with the use of prior information. The ARB of $\hat{\theta}_{(p,q)}$ is zero when $q = \lambda^{-1}$ (or $\lambda = q^{-1}$). This is a more realistic condition for unbiasedness without damaging the basic shrinkage type structure of $\hat{\theta}_{(p,q)}$. The MSE of $\hat{\theta}_{(p,q)}$ is derived as

$$\text{MSE}\left(\hat{\theta}_{(p,q)}\right) = \theta^2\left[(q\lambda - 1)^2\left(1 - W_{(n,p)}\right)^2 + \frac{W_{(n,p)}^2}{n}\right]$$

It is obvious that the MSE of $\hat{\theta}_{(p,q)}$ is minimum when $q = \lambda^{-1}$ (or $\lambda = q^{-1}$). The percent relative efficiency (PRE) of $\hat{\theta}_{(p,q)}$ with respect to $\tilde{\theta}_M$ is given by the formula:

$$\text{PRE}\left\{\hat{\theta}_{(p,q)}, \tilde{\theta}_M\right\} = \frac{n}{\left[n(q\lambda - 1)^2(1 - W_{(n,p)})^2 + W_{(n,p)}^2\right](n+1)} \times 100$$

The proposed class of shrinkage estimators $\hat{\theta}_{(p,q)}$ has smaller MSE than that of-

(i) UMVUE $\tilde{\theta}_U$, if-

$$\left(1 - \sqrt{G}\right)q^{-1} < \lambda < \left(1 + \sqrt{G}\right)q^{-1}$$

or equivalently, $$\left(1 + \sqrt{G}\right)^{-1} q\,\theta_B < \theta < \left(1 - \sqrt{G}\right)^{-1} q\,\theta_B.$$

Since the experimenter has confidence that $\theta \in (\theta_1, \theta_2)$, a more realistic condition may be $(\theta_1,\ \theta_2) \subseteq \left[\left(1+\sqrt{G}\right)^{-1} q\ \theta_B,\ \left(1-\sqrt{G}\right)^{-1} q\ \theta_B\right]$

where
$$G = \frac{1 - W_{(n,p)}^2}{n\left(1 - W_{(n,p)}\right)^2}.$$

(ii) MMSE estimator $\tilde{\theta}_M$, if-
$$\left(1-\sqrt{H}\right)q^{-1} < \lambda < \left(1+\sqrt{H}\right)q^{-1} \tag{3.3}$$

or equivalently,
$$\left(1+\sqrt{H}\right)^{-1} q\ \theta_B < \theta < \left(1-\sqrt{H}\right)^{-1} q\ \theta_B$$

or in practical situation
$$(\theta_1,\ \theta_2) \subseteq \left[\left(1+\sqrt{H}\right)^{-1} q\ \theta_B,\ \left(1-\sqrt{H}\right)^{-1} q\ \theta_B\right]$$

where
$$H = \left(1 - W_{(n,p)}\right)^{-2}\left[(n+1)^{-1} - W_{(n,p)}^2 n^{-1}\right].$$

The convex nature of the proposed statistic $\hat{\theta}_{(p,q)}$ and the condition of existence of gamma functions contained in $W_{(n,p)}$ provide the criterion of selecting the scalar p. Therefore, the acceptable range of value of p is given by

$$\{p \mid 0 < W_{(n,p)} < 1 \text{ and } p < (n/2)\}, \forall\ n.$$

The range of dominance of λ in (3.3) provides the criterion of selecting the scalar q. Therefore the acceptable range of value of q is given by

$$\left\{ q \mid \left(1-\sqrt{H}\right)\lambda^{-1} < q < \left(1+\sqrt{H}\right)\lambda^{-1} \right\}$$

The quantity λ in the above expression represents the average departure of the natural origin θ_B from the true value θ. But in practical situations it is hardly possible to get an idea about λ. Owing to this, an unbiased estimator of λ is proposed, namely
$$\hat{\lambda} = \left(\frac{n-1}{n}\right)\frac{\theta_B}{\bar{x}}.$$
Therefore replacing λ by $\hat{\lambda}$, the experimenter may choose q as
$$\left\{ q \mid \left(1-\sqrt{H}\right)\hat{\lambda}^{-1} < q < \left(1+\sqrt{H}\right)\hat{\lambda}^{-1} \right\}. \tag{3.4}$$

It is pointed out that at $q = \lambda^{-1}$, the proposed family of estimators $\hat{\theta}_{(p,q)}$ is not only unbiased but renders maximum gain in efficiency. This is a remarkable property of the proposed family of estimators. Thus in order to obtain significant gain in efficiency as well as proportionately small magnitude of bias for fixed λ, one should choose q in the vicinity of $q = \lambda^{-1}$. It is interesting to note that if one selects smaller values of q then higher values of

λ leads to a large gain in efficiency (along with appreciable smaller magnitude of bias) and vice-versa. This is legitimate for all values of p.

4. PERFORMANCE OF THE SUGGESTED FAMILY OF ESTIMATORS

So far we have dealt with the theoretical results. This section endowed with some numerical illustrations regarding the performance of the suggested family of Bayesian-shrinkage estimators. The ranges of q and λ in which the proposed estimator $\hat{\theta}_{(p,q)}$ is better (in the sense of efficiency) than the MMSE estimator $\tilde{\theta}_M$ have been reckoned and are displayed in Table 4.1. It has been observed that the range of q (or λ) shrinks down as the value of λ (or q) increases. In other words, if the crude Bayes estimate moves far from the true value, the length of the effective interval of q decreases. In fact, there is enough scope of choosing q to get better estimators in the class. Alternatively we can say that the range of λ is wider for q tending to zero and it abates for larger values of q.

Table 4.1 – Effective Range of Dominance of q (or λ) For p = -1 and Different Values of λ (or q).

Values of λ (or q)	Range of q (or λ)	Values of λ (or q)	Range of q (or λ)	Values of λ (or q)	Range of q (or λ)
0.05	(0 , 40.0)	1.50	(0 ,1. 33)	3.25	(0 , 0.61)
0.10	(0 , 20.0)	1.75	(0 , 1.14)	3.50	(0 , 0.57)
0.25	(0 , 8.00)	2.00	(0 , 1. 00)	3.75	(0 , 0.52)
0.50	(0 , 4.00)	2.25	(0 , 0. 88)	4.00	(0 , 0.50)
0.75	(0 , 2.66)	2.50	(0 , 0.80)	6.00	(0 , 0.33)
1.00	(0 , 2.00)	2.75	(0 , 0.72)	8.00	(0 , 0.25)
1.25	(0 ,1. 60)	3.00	(0 , 0.66)	10.0	(0 , 0.20)

To illustrate the performance of the suggested class of estimators $\hat{\theta}_{(p,q)}$ over MMSE estimator, PREs have been computed for several combinations of scalars involved in $\hat{\theta}_{(p,q)}$ and some of the findings are presented through Exhibits 4.1 and 4.2.

For fixed n, p and q, the gain in efficiency of $\hat{\theta}_{(p,q)}$ relative to MMSE estimator $\tilde{\theta}_M$ increases up to $\lambda = q^{-1}$ attains its maximum at this point and then decreases symmetrically in magnitude as λ increases in its range of dominance. Moreover, for fixed q and λ, the percent relative efficiency decreases as the sample size n increases. While comparing the two exhibits, the length of effective interval of λ is wider in case of $\hat{\theta}_{(-1,0.25)}$ as compared to $\hat{\theta}_{(1,0.25)}$. However, the gain in efficiency is more in case of $\hat{\theta}_{(1,0.25)}$ as compared to $\hat{\theta}_{(-1,0.25)}$.

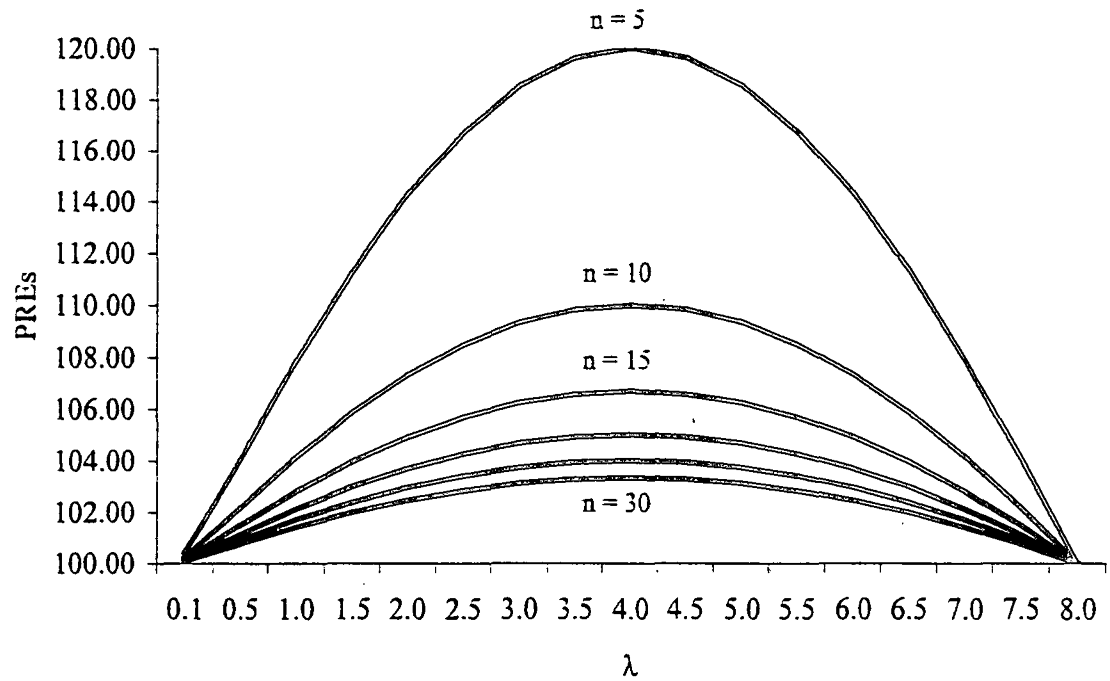

Exhibit 4.1. PREs of the proposed estimator with respect to MMSE estimator for n = 5(5)30, (p, q)=(-1, 0.25)

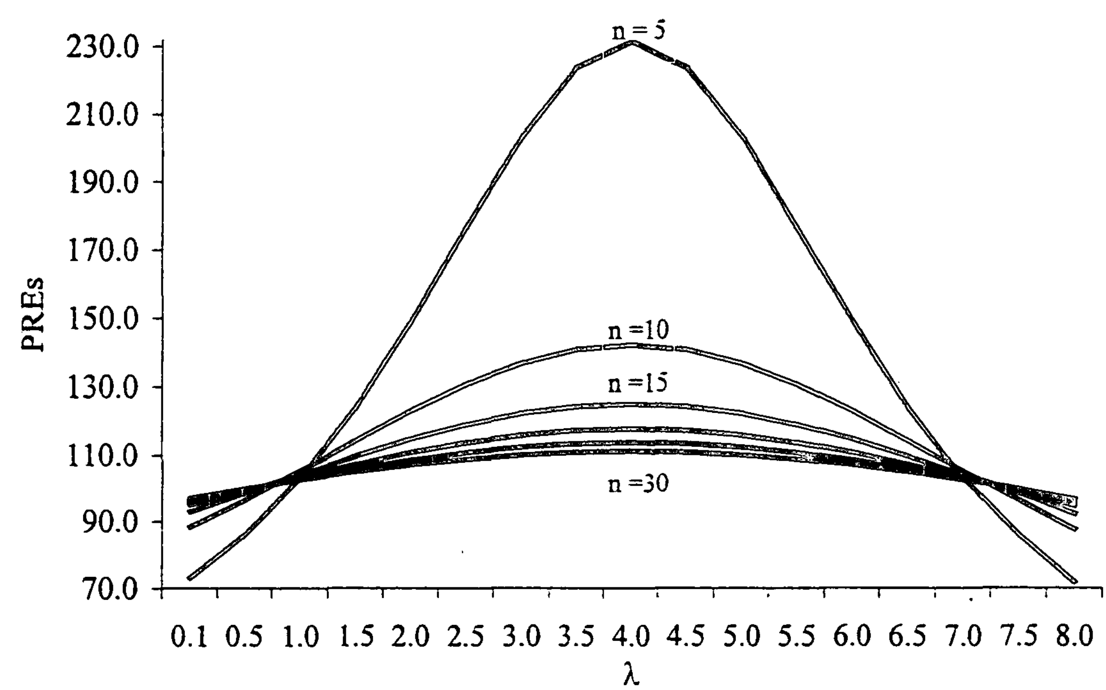

Exhibit 4.2. PREs of the proposed estimator with respect to MMSE estimator for n = 5(5)30, (p, q)=(1, 0.25)

5. AN APPLICATIVE EXAMPLE AND THE SUGGESTED FAMILY OF ESTIMATORS

To emphasize the application of the suggested class of shrunken estimators $\hat{\theta}_{(p,q)}$ an example of life-testing experiment is considered from Jani (1991). Ten electronic tubes were put to test and the test continued till of them failed. The failure times (in hours) were recorded as 273, 307, 344, 376, 415, 455, 502, 558, 619 and 681. Assuming that the sample has come from an exponential population defined by (1.1). Our interest is to estimate the mean life of the item. A prior point estimate of mean life θ is available from some similar study in the past as 450 hours and the experimenter is confident that the mean life θ does not fall outside the interval (410 , 520).

Here it is given that $n = 10$, $\theta_1 = 410$, $\theta_0 = 450$ and $\theta_2 = 520$. It is simple to calculate $\tilde{\theta}_U = \bar{x} = 453.00$ and $\tilde{\theta}_M = 411.81$. For an illustration, we have chosen $p = 2$ for which $W_{(n,p)} = 0.42$, although other choices of p may be possible. In order to obtain the Bayes estimate, it is relevant to calculate $\eta_1 = 0.0024$, $\eta_0 = 0.0022$, $\eta_2 = 0.0019$, $\theta_{mid} = 465 > \theta_0$, thus $\xi_1 = 2.40$ and $\xi_2 = 3.60$, $\mu = 2.20008 \times 10^{-3}$, $\alpha = 317065.12$, $\beta = 697.5686$, $\eta_B = 2.20018 \times 10^{-3}$, and finally, $\theta_B = 454.51$.

Table 5.1. –Impact of values of q on PRE

λ↓ \ q→	0.5867	0.6894	0.7921	0.8948	0.9975	1.1002	1.2029	1.3056	1.4083	1.5110	1.6137
0.3300	38.46	41.63	45.19	49.22	53.79	58.99	64.95	71.80	79.73	88.95	99.73
0.4385	44.74	50.12	56.49	64.09	73.25	84.37	98.01	**114.90**	**136.00**	**162.51**	**195.92**
0.5470	52.62	61.36	72.33	86.31	**104.39**	**128.08**	**159.52**	**201.46**	**256.93**	**327.43**	**408.23**
0.6555	62.68	76.59	95.29	**120.98**	**156.89**	**207.59**	**278.11**	**369.05**	**463.18**	**514.46**	**484.51**
0.7640	75.72	97.75	**129.72**	**177.17**	**247.84**	**347.08**	**458.07**	**515.30**	**464.29**	**354.28**	**253.36**
0.8725	92.97	**127.93**	**182.78**	**268.83**	**390.21**	**500.04**	**492.32**	**376.40**	**257.96**	**175.73**	**123.47**
0.9810	**116.19**	**171.89**	**264.57**	**401.13**	**510.87**	**460.18**	**318.48**	**205.85**	**136.48**	94.81	68.87
1.0895	**147.99**	**236.25**	**379.51**	**509.23**	**450.64**	**294.04**	**181.51**	**117.04**	79.96	57.49	43.09
1.1980	**192.03**	**326.58**	**490.94**	**469.03**	**298.73**	**175.95**	**109.42**	72.91	51.51	38.13	29.27
1.3065	**252.68**	**433.79**	**503.67**	**333.56**	**187.16**	**110.86**	71.28	49.09	35.65	26.99	21.10
1.4150	**332.50**	**509.53**	**403.13**	**219.23**	**121.87**	74.56	49.55	35.07	26.04	20.05	15.90
1.5235	**424.19**	**491.64**	**283.90**	**146.57**	83.79	53.03	36.24	26.22	19.80	15.46	12.40
1.6320	**498.19**	**396.90**	**195.66**	**102.24**	60.49	39.43	27.57	20.30	15.55	12.28	9.93
1.7405	**511.00**	**292.48**	**138.01**	74.42	45.47	30.38	21.65	16.17	12.52	9.98	8.13
1.8490	**453.20**	**211.29**	**100.73**	56.21	35.31	24.08	17.42	13.17	10.29	8.26	6.78
1.9575	**362.84**	**154.79**	76.01	43.78	28.15	19.54	14.32	10.93	8.61	6.96	5.73
2.0660	**277.36**	**116.25**	59.06	34.98	22.94	16.15	11.97	9.21	7.31	5.93	4.91
2.1745	**210.32**	89.62	47.05	28.54	19.04	13.57	10.15	7.87	6.28	5.12	4.26
2.2830	**161.19**	70.78	38.28	23.71	16.05	11.56	8.71	6.80	5.45	4.46	3.72
2.3915	**125.74**	57.10	31.71	19.99	13.70	9.96	7.56	5.93	4.77	3.93	3.28
2.5000	99.98	46.93	26.67	17.08	11.83	8.67	6.62	5.22	4.22	3.48	2.92

The possible range of q has been calculated as (0.5867, 1.6136) by virtue of expression (3.4) wherein $\hat{\lambda} = 0.9089$. Many choices of q in the range (0.5867, 1.6136) may be possible depending upon the level of risk the experimenter wants to take. If there are enough reasons to believe that θ_B underestimates θ then higher values of q should be chosen. As the degree of underestimation increases, $q \to 1.6136$ ensures larger gain in efficiency. On the other hand, if θ_B overestimates θ then smaller values of q should be chosen. As the degree of overestimation increases, $q \to 0.5867$ ensures larger gain in efficiency. If the experimenter is not having any idea of underestimation or overestimation but he is confident that the degree of underestimation/overestimation is moderate then middle values of q should be chosen in the interval of q, see Table 5.1.

In the present case, let us suppose that the experimenter has no idea about the estimate θ_B and he thus chosen $q = 1.05$ (say), the resulting Bayesian-shrinkage estimate is then given by $\hat{\theta}_{(2,1.05)} = 467.06$. Range of λ and θ in which $\hat{\theta}_{(2,1.05)}$ is better than $\tilde{\theta}_M$ in terms of efficiency are (0.5079, 1.3968) and (325.38, 894.85) respectively. Clearly, $(\theta_1, \theta_2) \subseteq (325.38,\ 894.85)$ and hence the proposed estimator $\hat{\theta}_{(2,1.05)}$ is always more efficient than $\tilde{\theta}_M$.

6. ESTIMATION WITH CENSORED SAMPLES

Hitherto we have assumed that we have a complete sample where the failure times of all the n items are recorded. For the case in which the failure-censored sample is available, the results follow from the preceding ones as much of the theory and derivation remains the same. Gamma$\left(\alpha + \sum_{i=1}^{r} x_i + (n-r)x_r,\ \beta + r\right)$ posterior would work for a crude Bayes estimator of θ. A class of Bayesian-shrinkage estimators for mean life θ is then obtained by merely replacing $\bar{x}$ by (S_r/r) and n by r in the family of estimators suggested in Section 3.

7. CONCLUDING REMARKS

The proposed family of estimators seems to be an intelligent use of point guessed value and guessed interval simultaneously. Graphs and numerical computations indicate that the proposed family of estimators substantially improves the UMVUE and it is better than the MMSE estimator even if the Bayes estimate of the scale parameter is far from the true value of the parameter. The family of estimators is effectively applicable when the sample size is small, as it is usually in the case of exponential distribution. If censored samples are considered the properties based on complete sample size n of the proposed class of estimators holds true for censored sample size r also no matter what is the value of n.

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